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QUANTUM ALGORITHMS WITH CONTROLLED HODGKIN-HUXLEY NEURONS

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Abstract

The dynamical system corresponding to the Hodgkin-Huxley (HH) neuron contains the control parameter, for instance, the electrical current or another external signal, stimulating the action potential (outcome) in the axon. Choosing the appropriate shape of the control via speed gradient or alternative algorithms one can keep the system imitating a quantum behaviour. The controlled four-dimensional HH system, in this case, involves effects similar to the quantum phase contributions to the computational process. We provide a simple example of the HH-based computational algorithm following the quantum paradigm. The linear chain of two HH neurons emulates the results of the Deutsch – Jozsa algorithm for the searching problem. To reproduce the output effect similar to the contribution of quantum phases the neurons are controlled by one of two alternative feedback versions: target attractor or speed gradient. We invent the successful classical emulation of the Deutch-type quantum algorithm and discuss the pros and cons of both

alternative feedback methods. Our approach can open a novel method for the practical realization of quantum algorithms and develop new perspectives for the computational properties of artificial neural networks (ANNs). The possible applications of the proposed algorithms are the modelling of epilepsy in the ANNs and the big data analysis.

Keywords

Hodgkin-Huxley Neuron, Quantum Computation, Deutsch – Jozsa Algorithm, Target Attractor Feedback, Speed Gradient Feedback

1. Introduction

The area of quantum informatics covers presently a wide range of computational algorithms based on the new ‘quantum logic’ paradigm. In parallel, the set of classical emulations of quantum algorithms starts to be the focus of research.

To model quantum computations, one needs to find a classical system with specific properties. First of all, it should be able to imitate efficient effects of the quantum phase contributions (Li & Lin, 2016). The system must be multi-dimensional in the phase space, and it has to demonstrate a variety of dynamical regimes. Nonlinear multidimensional dynamical systems presented in the form of ordinary differential equations with free control parameters cover a variety of regular and chaotic regimes and can be used for computational purposes and data analysis. To enter into a chaotic regime, for instance, the dimension of the phase space for the deterministic (i.e., not stochastic) continuous system should be at least 3 (Strogatz, 1994).

1.1. Literature Review

One of the first researches on the classical emulation of a quantum computation focused mostly on electronic signals has been done in (La Cour et al., 2016), where the classical signals of bounded durations and amplitudes have been used to represent the Hilbert space structure of a multi-qubit quantum state. Later on, the authors extended their model for the case of parallel computations (La Cour et al., 2018).

The complex network theory has been also inspired by quantum information methods. Here we can mention two types of quantum networks; for further details see the review (Biamonte et al., 2019). The first type consists of quantum systems with the connections represented by entangled states (Perseguers et al., 2010). These quantum networks are mostly used in quantum security and quantum communication areas. The second one is related to the networks representing the quantum physical systems: quantum particles and condensed matter

structures see, for instance, (Aspuru-Guzik & Walther, 2012). These networks cover a variety of physical prototypes: quantum dots (Lodahl, 2017), spin systems (Borregaard et al., 2019), solid-state qubits (Pompili et al., 2021).

Nevertheless, the research on the applications of networks to quantum algorithms focused mostly on emulating separate quantum logical operations and modelling the entangled states. Recently we discussed a novel approach for the imitation of quantum algorithm outcomes without the detailed modelling of individual logical gates (Borisenok, 2020a, b). The basic idea was to define the analogue of quantum states in the dynamical system via its distinctly different regimes (in the case of a neuron – via the resting/spiking alternative).

This model covers the gap of a simple output emulation of the quantum-paradigm-based algorithm without the detailed modelling of particular quantum gates.

1.2. Research Objectives

Here we discuss the dynamical system corresponding to the Hodgkin-Huxley (HH) mathematical neuron represented with four ordinary differential equations. Also, it contains a one-dimensional control parameter: the electrical current or another similar external signal, stimulating the action potential in the axon. We compare two multidimensional systems: classical 4-dimensional Hodgkin-Huxley (HH) neuron and a 3-dimensional quantum bit (in its real ODE representation) in the external field. Both systems can be driven via the free parameters towards the necessary dynamical state (stabilization or tracking goal).

We provide a simple example of the HH pair-based computational algorithms following the quantum paradigm. It is the Deutsch – Jozsa quantum procedure for a simple searching problem. We define the ‘states’ of the HH neuron emulating the pure qubit states, and propose a simple measurement procedure of resting or spiking of the following HH neuron for the searching problem in a single algorithmic cycle.

1.3. Scope of Study and Research Method

After Section 2 where we remind the basic properties of qubits and the main idea of the Deutsch – Jozsa algorithm, we introduce in Section 3 the mathematical model for the Hodgkin-Huxley neurons and their linear chains.

To demonstrate the possibility of using a multi-dimensional classical system for the realization of quantum algorithms we describe in Section 4 the HH-neuron based adaptation of the Deutsch algorithm with the feedback methods following two alternative approaches: Fradkov’s

speed gradient and Kolesnikov's target attractor feedback control and discuss the pros and cons of both versions to finalize the algorithm.

In Section 5 we conclude the results of our study, define constraints of the proposed algorithm, and discuss the future research perspectives.

2. Quantum Algorithms

To describe the basic features of quantum computations, let's briefly revive the mathematical approach to the main object: quantum bit (qubit). Sure, its evolution mostly follows von Neumann's equation for the density operator ρ , and it cannot be reduced to the classical description.

Nevertheless, under some simplification, we can present the dynamical system describing qubit with the set of real ordinary differential equations. If there is no decay due to the coupling of the qubit with the environment, it is represented via three variables.

2.1. Quantum Bits

In the case of 'ideal' qubit (with no decay), the system can be represented as a set of deterministic ordinary differential equations (Pechen & Borisenok, 2015):

$$\begin{aligned} \frac{dx}{dt} &= y; \\ \frac{dy}{dt} &= -x - u \cdot z; \\ \frac{dz}{dt} &= u \cdot y, \end{aligned} \quad (1)$$

where the functions could be expressed via the density operator elements:

$$\begin{aligned} x &= \rho_{12} \exp\{i\omega t\} + \rho_{21} \exp\{-i\omega t\}; \\ y &= i[\rho_{12} \exp\{i\omega t\} - \rho_{21} \exp\{-i\omega t\}]; \\ z &= \rho_{22} - \rho_{11}. \end{aligned} \quad (2)$$

Here the time is dimensionless, normalized in the units of the energy distance $\omega = E_2 - E_1$, where E_1 and E_2 are the energy ground and excited levels correspondingly; the Plank constant is chosen to be equal to 1.

The variables x and y in (1) correspond to the phase contribution, the variable z is the inversion, i.e., the difference of probabilities to find the qubit in two pure states. The dimensionless parameter u plays a role of a quasi-classical external control field.

It is easy to check that for any moment of time: $x^2 + y^2 + z^2 = 1$. Thus, all the evolution of the qubit takes place on the surface of a unity sphere (the ‘Bloch sphere’).

2.2. The Deutsch – Jozsa Algorithm

To give an example of a quantum algorithm, let’s choose the simple searching problem. Suppose that we get a function f mapping $\{0,1\}^n$ into $\{0,1\}$. According to the Deutsch approach, the function must be either constant or balanced:

1. If the function is a *constant*: the function $f(x) = 0$ for all x from $\{0,1\}^n$ or $f(x) = 1$ for all x from $\{0,1\}^n$;
2. If the function is *balanced*: the number of outputs 0 for the mapping is equal to the number of outputs 1.

The goal of the algorithm is to check if the given function f is a constant. To do it for the classical approach we need $2^{n-1} - 1$ evaluation. Quantum algorithms, from another hand, can perform it much faster.

The basic solution to this searching problem has been proposed by Deutsch in 1985 and generalized in 1992 in the form of the Deutsch – Jozsa algorithm for an arbitrary positive integer n (Deutsch & Jozsa, 1992). The circuit for the Deutsch – Jozsa algorithm is given in Fig.1. It contains three Hadamard gates, one two-qubit gate for the function f , and one measurement operation. The symbol \oplus stands here for the addition mod 2.

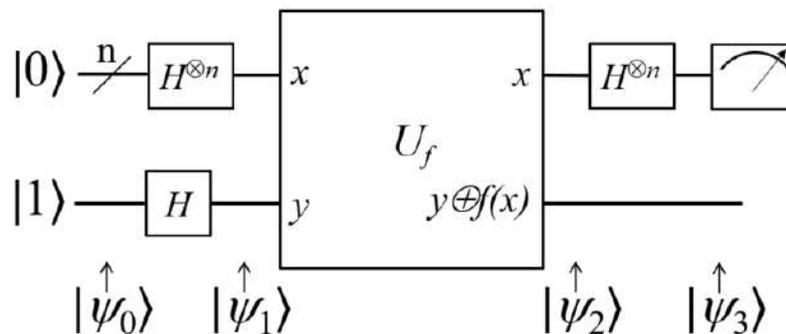


Figure1: *The Quantum Circuit for the Deutsch – Jozsa Algorithm*
 (Source: Aradyamath et al., 2019)

For simplicity, we will focus here on the case $n = 1$. Then the result of the measurement is equal to:

$$|\text{output}\rangle = \frac{1}{2} \left[(1 + (-1)^{f(0) \oplus f(1)}) |0\rangle + (1 - (-1)^{f(0) \oplus f(1)}) |1\rangle \right]. \quad (3)$$

Eq. (3) solves the problem of searching. Indeed, if $f(0) \oplus f(1) = 0$, then the output is $|0\rangle$, and the function f is constant. If $f(0) \oplus f(1) = 1$, then the output is $|1\rangle$, and the function f is balanced. Due to the so-called quantum phase kick-back effect in the algorithm, we need only a single measurement to distinguish between those two cases. This is exactly the feature of quantum algorithms: we do not need to make the sequent set of discrete operations for the function f , all such information is already included in the quantum phase as a result of a single measurement.

The same is valid for the case of n bits. If all n measurement results are $|0\rangle$, we conclude that the function was constant. Otherwise, if at least one of the measurement outcomes is $|1\rangle$, we conclude that the function was balanced.

The Deutsch-based family of quantum algorithms has a wide spectrum of applications: quantum computation, quantum cryptography, quantum data mining, simulation of quantum systems, modelling of formal languages (Batty et al., 2008).

3. The Hodgkin – Huxley Neuron

The problem now is how to develop a similar paradigm based on quantum computation and apply it to the classical system of Hodgkin–Huxley (HH) neurons.

The mathematical description of the HH neuron has been proposed by Alan Lloyd Hodgkin and Andrew Huxley in 1952 as a phenomenological model based on the experiments when the giant axon of the squid has been stimulated by the electrical current.

3.1. ODE Model for Hodgkin – Huxley Neuron

The Hodgkin-Huxley differential model has the set of four variables: the output membrane action potential $v(t)$ and three ion channel variables $m(t)$, $n(t)$, $h(t)$ related to the probabilities for the membrane gates to be open or closed (Hodgkin & Huxley, 1952):

$$\begin{aligned}
 C_M \cdot \frac{dv}{dt} &= -g_{Na} m^3 h \cdot (v - E_{Na}) - g_K n^4 \cdot (v - E_K) - g_{Cl} \cdot (v - E_{Cl}) + I(t) ; \\
 \frac{dm}{dt} &= \alpha_m(v) \cdot (1 - m) - \beta_m(v) \cdot m ; \\
 \frac{dn}{dt} &= \alpha_n(v) \cdot (1 - n) - \beta_n(v) \cdot n ; \\
 \frac{dh}{dt} &= \alpha_h(v) \cdot (1 - h) - \beta_h(v) \cdot h .
 \end{aligned} \tag{4}$$

The membrane variables m , n , h depends on the action potential v via the non-linear functions:

$$\begin{aligned}
 \alpha_m(v) &= \frac{0.1 \cdot (25 - v)}{\exp\left\{\frac{25 - v}{10}\right\} - 1} ; \beta_m(v) = 4 \cdot \exp\left\{-\frac{v}{18}\right\} ; \\
 \alpha_n(v) &= \frac{0.01 \cdot (10 - v)}{\exp\left\{\frac{10 - v}{10}\right\} - 1} ; \beta_n(v) = 0.125 \cdot \exp\left\{-\frac{v}{80}\right\} ; \\
 \alpha_h(v) &= 0.07 \cdot \exp\left\{-\frac{v}{20}\right\} ; \beta_h(v) = \frac{1}{\exp\left\{\frac{30 - v}{10}\right\} + 1} .
 \end{aligned} \tag{5}$$

The net external current $I(t)$ stimulating the axon is a control parameter in the HH model (4). The set of constants includes the potentials E_{Na} (equilibrium potential at which the net flow of Na ions is zero), E_K (equilibrium potential at which the net flow of K ions is zero), E_{Cl} (equilibrium potential at which the leakage is zero) in mV, the membrane capacitance C_M and the conductivities g_{Na} (sodium channel conductivity), g_K (potassium channel conductivity), g_{Cl} (leakage channel conductivity) in mS/cm²:

$$\begin{aligned}
 g_{Na} &= 120; E_{Na} = 115 ; \\
 g_K &= 36; E_K = -12 ; \\
 g_{Cl} &= 0.3; E_{Cl} = 10.36.
 \end{aligned} \tag{6}$$

The important property of the dynamical system (4) is the variety of regimes: it can demonstrate resting (the neuron does not show sufficient activity), spiking (the neuron produces a single spike), bursting (the neuron generates a series of spikes).

A particular dynamical regime depends on the input current I . For instance, if the current is below a threshold level, the HH neuron stays resting; if we overcome the threshold level, it generates a spike.

3.2. Linear Chain of the Hodgkin – Huxley Neurons

If we combine a few HH neurons in a linear chain, the output action potential of the previous cell defines the input of the following one. We use here our gain model for the transfer of the output signal from k -th neuron via its synapse towards the dendrite/soma input of the l -th neuron (Borisenok et al., 2018):

$$I_l(t) = \alpha \cdot [v_k(t) - v_{\text{rest}}] ; \alpha = \text{const} > 0, \quad (7)$$

with the phenomenological gain constant α . Here v_{rest} is the reference rest potential in the HH neuron.

For our algorithm, we use a linear chain of two HH neurons. The first cell plays the role of the computational element, while the second one works as a measuring element. For that we define the threshold (tr) level:

$$I_{\text{tr}} = \alpha \cdot (v_{\text{tr}} - v_{\text{rest}}) . \quad (8)$$

Now the output of the first neuron stimulates the particular regime of the second measuring element. If the first HH neuron produces the action potential below the threshold level v_{tr} , the second neuron does not spike. If the output action potential of the first neuron v achieves the threshold level, the second one produces a single spike.

4. Deutsch Algorithm with the Pair of Hodgkin – Huxley Neurons

Among the variety of algorithms for the feedback control, we consider here two alternatives. The first one is Kolesnikov's target attractor feedback forming in the phase space of a dynamical system an artificial attractor which locks the trajectories in its neighbourhood of the control goal subset. Another approach is based on Fradkov's speed gradient method leading the system towards the control goal by minimizing a non-negative differentiable target function.

4.1. 'Quantum States' of HH Neuron

To emulate the Deutsch algorithm, let's define the 'pure quantum states' for the HH neuron: the resting $|0\rangle$ and the single spiking $|1\rangle$:

$$\begin{aligned} |0\rangle &= 0 \cdot I_{tr} ; \\ |1\rangle &= 1 \cdot I_{tr} , \end{aligned} \quad (9)$$

which correspond to the action potentials:

$$\begin{aligned} v_{|0\rangle} &= v_{rest} + \frac{0 \cdot I_{tr}}{\alpha} = v_{rest} ; \\ v_{|1\rangle} &= v_{rest} + \frac{1 \cdot I_{tr}}{\alpha} = v_{rest} + \frac{I_{tr}}{\alpha} . \end{aligned} \quad (10)$$

To unify both cases, let's define the target potential via the CNOT logical operator over the function f :

$$v_* = v_{rest} + \text{CNOT}\{f(0), f(1)\} \cdot \frac{I_{tr}}{\alpha} . \quad (11)$$

The symbol * stands here for the potential v (the output of the first neuron) which should be the goal of our control signal I in (4).

Our network emulating the quantum algorithms consists of two sequent HH neurons (see Fig.2). The first neuron contains the information about the function f , and it is driven by one of the mentioned control algorithms towards the goal action potential (11).

The function f here is a theoretically defined arbitrary function that satisfies the Deutsch criteria: it is either balanced or constant (see Section 2.2).

The resulting action potential of the first neuron with the dendrite/soma model enters the second neuron, which plays the role of the measuring element.

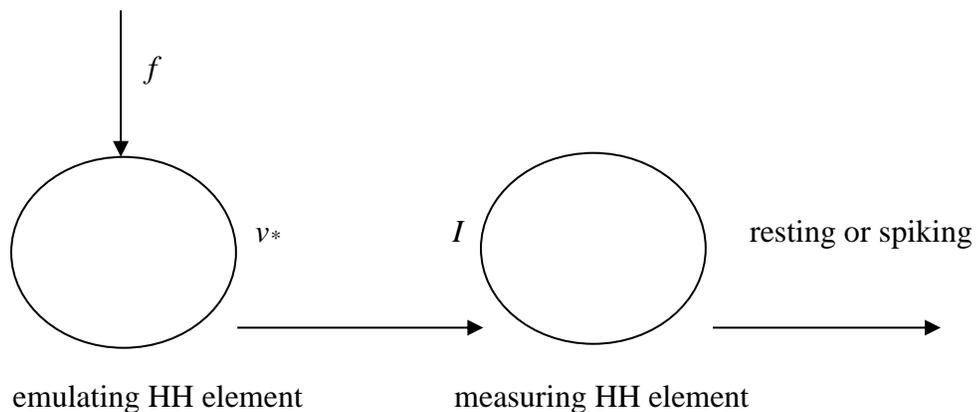


Figure 2: The Linear Chain of Two Hodgkin – Huxley Neurons Emulating the Deutsch – Jozsa Algorithm (Source: Self-Designed)

Now let's suppose that we can drive the first neuron towards the goal potential (11). Then it will stimulate the second measuring element with two different options: the second neuron will stay at rest or will generate a single spike. Based on it we can conclude if the function f is constant or not:

$$\begin{aligned} \text{If } f(0) = f(1), \text{ then } v_* = v_{\text{rest}} = v_{|0\rangle}, \text{ and } f \text{ is constant.} \\ \text{If } f(0) \neq f(1), \text{ then } v_* = v_{\text{rest}} + \frac{I_{\text{tr}}}{\alpha} = v_{|1\rangle}, \text{ and } f \text{ is balanced.} \end{aligned} \quad (12)$$

By (12) we reproduced the complete HH analogue of the Deutsch algorithm.

4.2. Target Attractor Feedback

Now the task is to provide the necessary output (11) for the first neuron. To do it, we can apply different control algorithms based on the gradient approaches (Fradkov, 2007) or the construction target attractors in the system, in the manner of 'synergetic' Kolesnikov's algorithm (Kolesnikov, 2013).

Let's start with Kolesnikov's approach. Let's define the target function:

$$\psi(t) = v(t) - v_*(t). \quad (13)$$

Minimizing the magnitude of (13) we drive our system towards the target potential (11). The control equation follows the control equation (Kolesnikov, 2013):

$$T \frac{d\psi(t)}{dt} = -\psi(t). \quad (14)$$

The positive constant T defines the typical time scale of the target attractor achievement.

Eq.(14) provides the exponentially convergent dynamics of the trajectories to the neighbourhood of the target attractor (13) in the phase space.

The control signal I for the first HH neuron is restored from the substitution of (13)-(14) into the first equation of the system (4). It becomes:

$$\begin{aligned} I_{\text{TA}} = C_M \cdot \left[\frac{dv_*}{dt} - \frac{1}{T}(v - v_*) \right] + g_{Na} m^3 h \cdot (v - E_{Na}) + \\ + g_K n^4 \cdot (v - E_K) + g_{Cl} \cdot (v - E_{Cl}). \end{aligned} \quad (15)$$

The achievability of the goal for arbitrary stabilization/tracking via target attractor feedback has been discussed in (Borisenok & Ünal, 2017).

4.3. Speed Gradient Feedback

To achieve the goal (11), alternatively let's use Fradkov's speed gradient algorithm (Fradkov, 2007). We define the non-negative differentiable goal function:

$$G(t) = \frac{1}{2} [v(t) - v_*(t)]^2. \quad (16)$$

It is dynamical (time-dependent), i.e. the control goal is tracking. The control signal for the Hodgkin – Huxley neuron is one-dimensional, for that reason the gradient is reduced to the partial derivative $\partial / \partial I$:

$$I_{SG} = -\Gamma \frac{\partial}{\partial I} \left(\frac{dG}{dt} \right) = -\frac{\Gamma}{C_M} (v - v_*) ; \Gamma = \text{const} > 0. \quad (17)$$

This control current (17) drives the system (4) towards the minimization of the control goal (16).

The achievability of the stabilization/tracking goal in the HH dynamical system for the speed gradient algorithm has been proved numerically in (Borisenok & Ünal, 2017).

Now the differential equation for the action potential v looks like this:

$$C_M \cdot \frac{dv}{dt} = -g_{Na} m^3 h \cdot (v - E_{Na}) - g_K n^4 \cdot (v - E_K) - g_{Cl} \cdot (v - E_{Cl}) - \frac{\Gamma}{C_M} (v - v_*) . \quad (18)$$

The produced action potential v enters the next HH neuron, in which we perform a 'measurement' by checking if it demonstrates the resting or spiking.

4.4. Comparison of the Alternative Feedback Approaches and Finalization of Algorithm

Now we are ready to finalize the Hodgkin – Huxley classical analogue of the Deutsch algorithm. Its sequent scheme is:

$$f \Rightarrow v_* \Rightarrow I \Rightarrow v = v_{|0} \text{ OR } v = v_{|1}. \quad (19)$$

We need to use a linear chain of two HH neurons. For the first neuron, we apply the control input I from (15) based on the goal action potential v_* from (11) defined over the function f . The potential v produced in the first neuron via the gain transfer (7) comes to the second neuron, i.e. to the measuring element. We detect if it stays in the rest or produces a spike. If we observe the resting, the function f is constant. If it spikes, f is balanced.

The choice of the particular feedback method, target attractor vs speed gradient, depends on the practical applications. In general, the gradient-based algorithms are less energy-consuming,

and they could be easily computed in the real-time regime. On the other hand, they are less accurate in the achievement of the goal compared with target attractor feedback. The main handicap of the target attractor method is a permanent pumping of the energy to the system via the control field.

Our algorithm (19), as in the case of the Deutsch approach (1), demands only a single measurement based on the function f .

We need to emphasize that here, like in the case of the quantum algorithm, for the constant case of the function f we cannot learn what the exact output of the function is: '0' or '1'.

5. Conclusions

We provided here a simple example of the HH-based computational algorithms following the quantum paradigm.

We proved that the linear chain of two controlled 4-dimensional Hodgkin – Huxley dynamical elements is capable of imitating the output effects similar to the quantum phase contributions during the computational process.

5.1. Results

Our control algorithm is robust, it does not depend on the initial state of the HH neuron, and it is stable under the relatively small external perturbation and noise.

Our algorithm involves two HH neurons, the first one works as an analogue of the whole Deutsch – Jozsa quantum circuit in Fig.1, the second HH element in Fig.2 serves as a measurement device. As in Deutsch – Jozsa's quantum algorithm, we need only a single measurement to solve the searching problem for the function f .

The algorithm proposed here could be easily generalized for the case of a few HH neurons in small ANNs (the classical analogue of the multi-qubit system).

Of course, for time optimization, quantum algorithms work faster than their classical emulations. Nevertheless, the HH-based classical algorithms are easier for practical implementation and do not demand very sophisticated software.

Thus, in summary, our approach can open a new gate for the practical realization of quantum algorithms and provide novel perspectives for the computational properties of artificial neural networks. Such ANN must include the neurons with the high dimension of their phase space, like in the case of Hodgkin – Huxley elements.

5.2. Constrains of the Proposed Algorithm

The classical emulations of quantum algorithms can be performed not only with the 4d differential HH system. Other classical differential models, for instance, FitzHugh – Nagumo neurons may play a similar role (Borisenok, 2021).

The multi-dimensional non-linear dynamical system which is taken for the emulation of the Deutsch-type algorithm must have at least two distinctly different dynamical regimes driven by external parameters, like resting vs spiking controlled by the external current stimulating the axon in the HH neurons.

5.3. Future Research Perspectives

In our study, we focused mostly on the Hodgkin – Huxley elements because in perspective we plan to adopt the Deutsch-type algorithm to the detection and suppression of the epileptiform regime in ANNs to model epilepsy in the human cortex. It will be a matter of our further research. The HH system is the closest one to mimic the real behavior of biological neurons.

Another possible application of the proposed algorithm is developing small-scale classical ANNs for big data analysis (Borisenok, 2020c). They also could be based on the Hodgkin – Huxley or the FitzHugh – Nagumo neurons.

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